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Economic Order Quantity (EOQ) for Perishable Goods with Weibull Distribution and Exponential Demand Rate Proportional to Price*

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Abstract

Business organizations that deal with consumable and perishable items have consistently incurred enormous loss as a result of the nature of their goods. The losses have direct negative impact on revenues. Unplanned and lack of precise production prediction models are responsible for this. An appropriate prediction model, developed to guide production plan and processes will help manufacturers in deciding which product to make and in what quantity. In this study, the Economic Order Quantity (EOQ) for perishable goods with Weibull lifetime distribution and exponential demand rate proportional to price was developed for perishable goods. The differential equations governing the instantaneous state of inventory in the interval [0, t_2] were obtained and solved for the equation of the quantity of inventory at time t. Using fixed parameters for the weibull and exponential distributions, simulation study was conducted on the derived EOQ model using R programming language. The simulation shows that the EOQ increases with increase in Weibull parameter. Real data on six loafs of bread obtained from Afe Babalola University bakery was used to illustrate how the model works. Result shows a good fit to the data and the average EOQ ranges from 60 to 400 loafs with ordering times of either 1 or two days interval. The pattern of EOQ varies between type of loafs of bread. The EOQ model developed is shown by this result to be appropriate for perishable goods with weibull lifetime distribution and exponential demand rate proportional to price.

Keywords: Economic order quantity, exponential demand rate, Perishable goods, weibull distribution.

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1. Introduction

Inventory remains one of the vital aspects of production and storage in companies and organizations. Manufacturing, materials, and marketing departments are three of the major operating sections of an organization, whether it is a production or a service organization. They have to maintain some level of inventory for smooth running of the business. Costs are incurred when large amount of resources are kept in inventory for a long time. However there are cases where perishable or deteriorating products are stored in inventory. These products depreciate with time. A perishable product is a product in which its quality deteriorates due to environmental conditions through time. In other words, the longer time the product spends in inventory the lower the worth of the product becomes until it gets to a time when its value depreciates to nothing. Deterioration of products in inventory is a frequent occurrence and a natural phenomenon which needs attention. In reality, the life cycle of seasonal product such as fruits and others like electrical components, volatile liquid and so on are short and they certainly have similar life span, that undergo gradual depreciation. Thus, the item may not serve the purpose after a period of time and will have to be discarded as it cannot be used to satisfy the future demand of customers (Sanje and Neeraj 2016).
Inventory control has a very important role since most organizations tend to allocate large amount of investments in their inventory. The addition of extra products to inventory will elevate the holding cost and capital. Extra inventory will result in extra cost of inventory and maintenance. Extra inventory will also put some of the company's capital into a stagnant state and cause a loss in opportunity cost. On the contrary, fewer inventories might lead to a stock out and urgent purchases will cost higher than the normal order.

Modeling inventory is one of the most important concepts in logistics. Having a solid strategy could save company lots of money and also might be the difference between not being able to compete against the competitors and having a famous position in the market sector. One major importance of inventory modeling is the ability of inventory models to predict the quantity of demand of a particular product. If it would be known in advance the point at which a certain good will be sold, inventory management would be a lot simpler. However, planning becomes more complicated when demand for commodity changes with price while depreciation rate change with time. This is a challenge companies need to deal with and which is addressed in this research. Companies tend to keep their stock filled enough to reduce the amount of lost sales, but storage of redundant goods costs both time and money. Since having an adequate balance between these two aspects is essential, it is vital to find the optimal strategy of how much to replenish and at which moment. The optimal strategy depends on the trade-off between lost sales and how much storage cost in keeping the inventory.

The rest of the article is arranged as follows: Section two is literature review, section three discussed the model, the distributions and the derivation of the EOQ, section four presents the simulation study and the application to real data and finally section five is the conclusion.

1.1 Literature Review

Several researches have been conducted on non-perishable goods. Researchers have done concrete work on perishable goods with linear and quadratic demand rates but adequate attention have not been given to modeling inventory of perishable goods with demand rate changing with price.

Harris (1915) was the first to develop inventory model, Economic Order Quantity, which was later generalized by Wilson (1934) who gave a formula to obtain economic order quantity. Wagner and Whitin (1958) considered the deterioration of the fashion goods at the end of the prescribed shortage period. Wagner and Whitin (1958) developed a dynamic version of the classical Economic Order Quantity model (EOQ) for deteriorating items.

Ghare and Schrader (1963) were the first to use the concept of deterioration, they developed an inventory model with a constant rate of deterioration, followed by Covert and Philip (1973) who formulated a model with a variable rate of deterioration with two parameter Weibull distribution, Philip then generalized the model by considering a three-parameter Weibull distribution deterioration with no shortages and a constant demand (1974), which was further extended by Shah and Jaiswal (1977). Dave and Patel (1981) studied an inventory model with deterministic but linearly changing demand rate and constant deterioration over a finite planning horizon. Hollier and Mak (1983) had developed two mathematical models for an inventory system in which the units are deteriorating at a constant rate and the demand rate which follows negative exponential decrease. Sachan (1984) extended Dave and Patel's model to allow shortages. Datta and Pal (1990) developed an EOQ model by introducing a variable deterioration rate and power demand pattern. T.C.E. Chang (1989) had considered infinite-horizon inventory problem in which items are decaying at a constant rate and the demand rate follows an exponentially decreasing function. Heng et al. (1991) proposed an exponential decay in inventory model for deteriorating items by assuming a finite replenishment rate and constant demand rate.

Chung and Ting (1994) determined the replenishment schedules for deteriorating items with time proportional demand. Hariga and Benkherouf (1994) developed an inventory replenishment model for deteriorating items with exponentially varying demand. This work was extended by Hariga (1995) to allow shortages, he developed models for deteriorating items with time dependent demand. Chakrabarti and Chakrabarti and Chaudhuri (1997) studied an inventory model with linearly changing demand rate, constant deterioration rate and shortages in all cycles over a finite-planning horizon. Bhunia and Maiti (1998) had formulated and solved two models for deteriorating items with linearly time dependent demand where replenishment rate changes linearly with the change in demand and on-hand inventory amount. Jalan and Chaudhuri (1999) presented their model taking exponentially time dependent demand rate. Wu (2001) considered an inventory model with Weibull distribution deterioration and ramp type demand rate in which shortages are allowed and the backlogging rate is dependent on waiting time. Giri et al. (2003) extended the ramp type demand inventory model with a more generalized Weibull deterioration distribution. Manna and Chaudhuri (2006) developed an inventory model for time-dependent deteriorating items with ramp type demand rate. Skouri et al. (2009) considered an inventory model with general ramp type demand rate, partial backlogging, and Weibull deterioration rate. Singh and Banerjee (2010) proposed an economic order quantity model for deteriorating items having stock dependent demand under the effect of inflation. Hung (2011) extended Skouri et al. (2009) inventory model from ramp type demand rate and Weibull deterioration rate to arbitrary demand rate and arbitrary deterioration rate. Muriana (2016) developed a mathematical stochastic model for perishable open-dating foods including shortage and outdating costs. The demand fluctuations were taken into account through a normal distribution, and their impact on the storage were. The determination of perished products was addressed, determining the probability for a product of remaining in stock beyond the end of its Shelf.

The basic EOQ model assumes a constant demand rate and an infinite planning horizon, there is no deterioration of inventory and replenishment is instantaneous i.e. the lead time is zero. These assumptions restrict the applicability of the classic Economic Order Quantity (EOQ) model. In order to make the basic model more realistic, many researchers have extended the Wilson's Economic Order Quantity (EOQ) model by considering time varying demand pattern and deterioration rate. Singh and Banerjee (2019) considered perishable items whose deterioration starts immediately after procurement with constant rate of deterioration. The goods considered were fast-moving goods whose demand is increasing at a very rapid pace and the result provides buyers with a policy that aids them to decide their optimal order quantity. Díaz et al. (2020) considered situations where demand and sales do not go hand-in-hand. They also considered the cost of disposing of unsold units, besides the conventional cost of storage and procurement. Patriarca et al. (2020) present an inventory control model for perishable items with a demand rate variable over time and dependent on the inventory rate. It is germane to note that the depletion of any inventory is due mainly to the combined effects of demand and deterioration of the item.

This paper, determines the economic order quantity for perishable goods with Weibull lifetime distribution and exponential demand rate proportional to price. Weibull is popularly known for modeling survival rates and the exponential distribution is a special case of the Weibull distribution. To the best of our knowledge, the modeling structure adopted in this paper has not been previously conducted. Allowing exponential decay for the demand rate against price relaxes the assumption that the price of a good is fixed throughout its lifetime. For a fast perishable good, the decay parameter is chosen such that the price for the first few days after production is approximately kept constant.

2. Metodologi

The probability density function of a Weibull distribution is given as

$$f(x) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}, \text{ for } x \geq 0 \tag{2.1}$$

$$and\ f(x) = 0, \text{ otherwise.}$$

The mean is $\lambda\Gamma(1 + 1/k)$ and the variance is $\lambda^2\left[\Gamma\left(1+\frac{2}{k}\right) - \left(\Gamma\left(1+\frac{1}{k}\right)\right)^2\right]$. The hazard function can be written as $H(x;k,b) = bkx^{k-1}$ and the probability density function at time t becomes

$$F(t;k,b) = bkt^{k-1}e^{-bt^k} \tag{2.2}$$

with mean given as $b^{-\frac{1}{k}}\Gamma(1+\frac{1}{k})$.

An ordered level inventory model with a finite rate of replenishment is developed with the following assumptions.

(a) Exponential demand rate given by

$$f = d(p) = ae^{bp}\ ,\ \ p \geq 0, a, b \geq 0 \tag{2.3}$$

where a, is the initial rate independent of price and b is the rate with demand on price p. In other words on the long run, demand for commodity decreases exponentially with price.

(b) The production rate given by;

$$K = rd(p) \tag{2.4}$$

where, $r > 1$. The higher the demand the more quantity is being produced and stored in inventory.

(c) The deterioration of Inventory per unit of time is given as

$$\theta(t) = \alpha\beta t^{\beta-1} \tag{2.5}$$

where, $0 < \alpha \ll 1$ is the scale parameter, $\beta \geq 1$ is the shape parameter and , $t \geq 0$.

(d) The lead time is zero,

(e) The unit production cost v is inversely related to the demand rates as;

$$v = \alpha_1 d(p)^{-\gamma} \tag{2.6}$$

where $\gamma = 2, \alpha > 0$.
The rate of change of production cost with demand rate is obtained by differentiating v with respect to f.

2.1 Model Formulation

Let $Q(t)$ be the inventory level at time t($0 \leq t \leq t_2$). The instantaneous state of inventory in the interval [0, t_2] is governed by the differential equations

$$\frac{d(Q(t))}{d(t)} + \theta(t)Q(t) = K - d(p) \qquad 0 \leq t \leq t_1 \tag{2.7}$$

$$\frac{d(Q(t)}{d(t)} + \theta(t)Q(t) = -d(p) \qquad t_1 \leq t \leq t_2 \tag{2.8}$$

The stock level is initially zero at time $t = 0$. Production begins just after $t = 0$ and continues till $t = t_1$ and stop as soon as the stock level becomes S. Then the inventory level decreases due to demand and deterioration till it becomes zero at $t = t_2$
The solution of equations (3.7) and (3.8) using conditions that $Q(t_1) = S \; and \; Q(t_2) = 0$ is derived as; (See Appendix I)

$$Q(t) = a(1-r)\left[(t_1 - t) + bp(t_1 - t) + \frac{\alpha}{\beta+1}(t_1^{\beta+1} - t^{\beta+1})\right] - ea(1-r)\left[(t_1 - t) + bp(t_1 - t)\right]t^{\beta}, 0 \leq t \leq t_1$$

$$Q(t) = a\left[(t_1 - t) + bp(t_1 - t) + \frac{\alpha}{\beta+1}(t_1^{\beta+1} - t^{\beta+1})\right] - a\alpha\left[(t_1 - t) - bp(t_1 - t) + \frac{\alpha}{\beta+1}(t_1^{\beta+1} - t^{\beta+1})\right]t^{\beta}, t_1 \leq t \leq t_2$$

2.2 Simulation

The simulation was carried out using R programming language. The exponential and Weibull distribution parameters were fixed at different intervals to determine the behavior of $Q(t)$ in different scenarios. The fixed parameters were chosen at random. Table 2.1 present a summary of chosen parameter values.

Table 2.1 Parameter Values for the Two Chosen Distributions

S/N	K	$Lambda$	S/N	K	$Lambda$
1	0.5	0.5	9	0.5	1.5
2	1.0	0.5	10	1.0	1.5
3	1.5	0.5	11	1.5	1.5
4	2.0	0.5	12	2.0	1.5
5	0.5	1.0	13	0.5	2.0
6	1.0	1.0	14	1.0	2.0
7	1.5	1.0	15	1.5	2.0
8	2.0	1.0	16	2.0	2.0

The simulation results are shown from Figure A.1 to A.16 in Appendix II. Each graph contains three panels. On the left panel, it shows the economic order quantity at different values of the exponential distribution *0.5* (green), *1.0* (black), *1.5* (blue), *2.0* (red), while the middle panel shows the demand against unit price and the right panel shows the daily sales against time (hour). Simulations studies were conducted to examine the behavior of the EOQ models at different parameter of the Weibull scale and shape parameters. Higher parameters signifies fast rate of deterioration. Together with low rate of demand will lead to low EOQ whereas low parameter with low rate of demand gives low EOQ. This implies that lower quantity should be stocked in the inventory when there is high rate of deterioration. As the Weibull parameter grows and the exponential parameter grows the EOQ will be low.

3. Results and Discussion

Data set for this study was obtained from Afe Babalola University Bakery, Ado Ekiti, Nigeria. The data is a secondary data and it is the record of supply for various retailers (customers) from January to July 2019. It is obvious that loafs of bread deteriorate within a short time depending on the processing method. The deteriorating rate was modeled using exponential distribution. As seen in the simulation in section two, the higher the parameter value lambda, the faster it deteriorates and eventually levels off. In this section, the EOQ derived in section two was applied to the data.

There are six types of breads that are of interest which exhibits different deterioration rates. The quantity demanded and days were observed. The data are presented in Appendix III. The Weibull model was fitted on the data to determine the parameters that sufficiently model the data. This was computed using easy fit

3.1 Weibull Model of the data using Easy Fit

The Weibull distribution fitted the observed data satisfactorily (Figure 3.1). The densities for each bread type were lower than 0.5. The density demonstrated two patterns. In some products, such as Family size bread, Big Sardine, and Fast roll bread, the density starts off from the top and slowly depreciates until it attain its lowest density. Whereas in Small size, Small Sardine, Brico Bread, the density of the Weibull starts off at low density and then appreciates until it gets to the peak and slowly depreciates until it attains its lowest density. It represents the time until the breads are no longer edible. Breads with longer depreciation time have higher density.

[Figure 3.1 to be placed Here]

3.2 Goodness of fit for the Distribution

Three types of fitness evaluations were computed; Kolmogorov Smirnov, Anderson Darling and Chi-Square. The parameters obtained from distribution fits were used to evaluate EOQ over time. Table 3.1 presents the goodness of fit of Weibull distribution. Judging by Kolmogorov goodness of fit, the Weibull fit to Big Sardine is the best. Whereas, judging by Anderson darling, Brico has the best fit. Small size bread has the best fit when making decision based on Chi square. Table 3.2 presents the ranks of the goodness of fits. It can be concluded from the total rank that small size has the best fit since it has the lowest total rank. This conclusion agrees with the graphical fits on Figure 3.1.

Table 3.1: Goodness of fit of Weibull Distribution

	Kolmogorov Smirnov	Anderson Darling	Chi Squared
Family Size	0.14516	10.442	49.79
Small Size	0.10698	14.082	16.006
Big Sardine	0.08942	15.967	23.2
Small Sardine	0.15663	29.953	28.793
Brico	0.12313	10.37	38.481
Fast Roll	0.13674	25.686	49.511

Table 3.2: Ranks of the goodness of fit of Weibull Distribution

	Kolmogorov Smirnov	Anderson Darling	Chi Squared	Total
Family Size	5	2	6	13
Small Size	2	3	1	6
Big Sardine	1	4	2	7
Small Sardine	6	6	3	15
Brico	3	1	4	8
Fast Roll	4	5	5	14

3.3 Determination of the Economic Order Quantity

Figure 3.2 presents the Economic order quantity of the different types of bread. It can be seen graphically that the Weibull distribution appropriately fits the data. The Economic orders were plotted against the number of days before inventory replenishment. As the number of days increase, the EOQ increases, forming the pattern of a distribution function. The model suggests that the EOQ for Family size bread should be 400 loafs every day in order to satisfy the customers and reduce losses. The inventory is to be replenished after two days with a maximum of 300 loafs for Small size bread. Inventory for Big Sardine should be replenished every two days with about 150 loafs of bread. This also applies to Small Sardine but 200 loafs of bread should be produced. For Broco and Fast Roll, 80 loafs and 60 loafs of bread respectively should be produced daily. The EOQ for the six types of Bread is shown in Table 3.3.

Table 3.3: EOQ for Six Types of Bread**.**

Type of Bread	Economic Order Quantity (EOQ)	Ordering Time (Days)
Family Size	400	1
Small Size	300	2
Big Sardine	150	2

Small Sardine	200	2
Broco	80	1
Fast Roll	60	1

4. Conclusion

The Weibull distribution was assumed for the rate of deterioration with exponential demand rate proportional to the price of product. Weibull distribution can model times until products decay to the point where it is no longer consumable. Exponential distribution was used to model the rate at which customers demand the products. Two differential equations were solved for the EOQ. The EOQ determines the quantity to stock in the inventory considering only product demands and deterioration rate, and other variables were kept constant. Simulations studies were conducted to examine the behavior of the EOQ models at different parameter of the Weibull scale and shape parameters. We observed that higher parameters signifies fast rate of deterioration whereas low parameter start off at low points and slowly decays to zero. The effect on the EOQ was equally determined. With high parameters; the EOQ drops exponentially which implies that lower quantity should be stocked in the inventory when there is high rate of deterioration. This implies that as the Weibull parameter grows and the exponential parameter grows the EOQ decreases.

The model was applied to a bread production facility and it was found to be appropriate. EOQ was obtained for the various types of bread produced at the facility, together with the ordering times (in days). The EOQ ranges from 400 loaves (for family size bread) down to 60 (for fast roll bread).

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